SAGE Algorithm Based Channel Estimation for Uplink STBC-MC-CDMA Systems

Yusuf Acar†, Arshad Salih‡ and Hakan Doğan†
†Department of Electrical and Electronics Engineering, Istanbul University, Avcilar 34850, Istanbul, Turkey
hdogan@istanbul.edu.tr, ersat-turkmeneliden@hotmail.com.tr
‡Department of Electronics Engineering, Istanbul Kültür University, Bakirkoy, 34156, Istanbul, Turkey
y.acar@iku.edu.tr

Abstract—In this work, maximum likelihood (ML) channel estimation for uplink space time block coded multicarrier code-division multiple-access (STBC-MC-CDMA) systems is considered in the presence of frequency selective channel. It is shown that the ML channel estimation requires matrix inversion and its calculation requires significant computation for increasing of total active users and the length of channel. Therefore, the space-alternating generalized expectation-maximization (SAGE) algorithm is introduced to achieve the same performance with the ML channel estimation. We compared SAGE algorithm in terms of the number of used iteration and show that the proposed algorithms converge the same performance of the ML estimator as the increasing number of iterations while it requires significantly lower computational complexity.

Index Terms: MC-CDMA Systems, STBC, Channel estimation, Maximum Likelihood, SAGE.

I. INTRODUCTION

Multicarrier Code-Division Multiple Access (MC-CDMA) is a promising approach that marries the best of the OFDM and CDMA techniques [1]. It provides high data rate for frequency selective channels while allowing the system to support multiple users at the same time [2]. MC-CDMA systems can additionally employ multiple-input and multiple-output (MIMO) techniques to increase the performance of communication [3]. If the channel state information (CSI) is not known at the transmitter, the transmit diversity methods such as space-time block coding (STBC) which uses coding across antennas and time can be employed [4]. Alamouti proposed a very simple transmitter diversity technique for two Tx antennas, which provides a full diversity order, has no loss of capacity [5].

In this paper, a channel estimation problem at the receiver for the uplink STBC-MC-CDMA systems is considered. The channel estimation problem for MC-CDMA systems is investigated in [6] and the least square (LS) technique is applied for a single user system. For the uplink problem, since the received signals are a superposition of signals transmitted from different user antennas, the simple channel estimation techniques used in single user systems cannot be used. Therefore, in this paper, we firstly proposed the maximum likelihood (ML) channel estimator to estimate the channel by using pilot tones. In the ML solution, a matrix inversion is necessary to estimate channel variation. The complexity of the matrix inversion is increasing by the increasing number of subcarriers, antennas and users.

The expectation-maximization (EM) algorithm [7] is an iterative approach which converges the maximum-likelihood (ML) estimate whose direct calculation is computationally prohibitive. Several iterative procedure based on EM algorithm were applied to channel estimation problem [8], [9]. EM is also considered for the channel estimation in OFDM receivers and compared with SAGE version [10].

To solve complexity problem in the ML channel estimation, the Space-Alternating Generalized Expectation Maximization (SAGE) algorithm is proposed for the channel estimation. In the complexity section, it is demonstrated if the SAGE algorithm is applied, then the computational complexity of the channel estimation is lower than ML channel estimation due to not require matrix inversion.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; (·)∗, (·)T, (·)† and (·)−1 denote the conjugate, transpose, conjugate transpose and matrix inversion respectively; ∥·∥ denotes the Frobenius norm; I denotes the L × L identity matrix.

II. SIGNAL MODEL

This section we consider a baseband STBC-MC-CDMA uplink system with P sub-carriers, K mobile users which are simultaneously active, two antennas in the transmitter and one receive antenna in the receiver. The extension to two receive antennas is simple. The block diagram for the MISO STBC-MC-CDMA system is illustrated in Figure 1. For the kth user, after STBC coding each transmit symbol is modulated in the frequency domain by means of a P × 1 specific spreading sequence c_k. Then transforming by a P-point IDFT and parallel-to-serial (P/S) conversion, a cyclic prefix (CP) is inserted of length equal to at least the channel memory (L). In this paper, to simplify the notation, it is assumed that the spreading factor equals to the number of
sub-carriers and all users have the same spreading factor. Consequently, the signal is transmitted through a multipath channel with impulse response

\[ g_k(t) = \sum_{l=1}^{L} g_{k,l} \delta(t - \tau_{k,l}) \]  

where \( L \) is the number of paths in the \( k \)th users channel; \( g_{k,l} \) and \( \tau_{k,l} \) are, respectively, the complex fading coefficient and the delay of \( l \)th path and \( P_k \) is the transmit power of the \( k \)th user. The fading process is assumed to be white. Note that the \( L \)-dimensional discrete channel impulse response vector \( \mathbf{g}_k = [g_{k,1}, g_{k,2}, \cdots, g_{k,L}]^T \) and the transmission power \( P_k \) can be combined as \( \mathbf{h}_k = \sqrt{P_k} \mathbf{g}_k \), since they can not be separated from each other. It is assumed that the channel coefficients do not change along a symbol, but they change symbol to symbol.

At the receiver, the received signal is sampled at chip-rate, serial-to-parallel (S/P) converted, CP is removed, and DFT is then applied to the discrete time signal to obtain the received vector expressed as (for \( n = 1, 3, 5, \ldots, (N - 1) \))

\[ y(n) = \sum_{k=1}^{K} [s_k(n) \mathbf{C}_k \mathbf{F} \mathbf{h}_k^1 + s_k(n+1) \mathbf{C}_k \mathbf{F} \mathbf{h}_k^2] + \mathbf{w}(n) \]  

\[ y(n+1) = \sum_{k=1}^{K} [-s_k^*(n+1) \mathbf{C}_k \mathbf{F} \mathbf{h}_k^1 + s_k^*(n) \mathbf{C}_k \mathbf{F} \mathbf{h}_k^2] + \mathbf{w}(n+1) \]

where \( s_k(n) \) denotes data sent by the user \( k \) within the \( n \)th symbol; \( \mathbf{h}_k^m \) is the channel that between the \( k \)th user’s \( m \)th transmitter antenna and receiver antenna; \( \mathbf{C}_k = \text{diag}(\mathbf{c}_k) \) with \( \mathbf{c}_k = [c_{k1}, c_{k2}, \ldots, c_{kP}]^T \) where each chip, \( c_{ki} \), takes values in the set \( \{-\sqrt{P_k}, \sqrt{P_k}\} \) denoting the \( k \)th users spreading code; \( \mathbf{F} \in \mathbb{C}^{P \times L} \) denotes the DFT matrix with the \((k,l)\)th element given by \( e^{-j2\pi kl/P} \); and \( \mathbf{w}(n) \) is the \( P \times 1 \) zero-mean, i.i.d. complex Gaussian vector that models the additive noise in the \( P \) tones, with variance \( \sigma^2/2 \) per dimension.

We suppose \( N \) symbols are transmitted and stack \( \mathbf{y}(n) \) as \( \mathbf{y} = [y^T(1), \ldots, y^T(N)]^T \). By using the following equations,

\[ \mathbf{s}_a = \begin{bmatrix} s_1(1) \mathbf{C}_1 \mathbf{F} & s_2(1) \mathbf{C}_2 \mathbf{F} & \cdots & s_K(1) \mathbf{C}_K \mathbf{F} \\ s_1(3) \mathbf{C}_1 \mathbf{F} & s_2(3) \mathbf{C}_2 \mathbf{F} & \cdots & s_K(3) \mathbf{C}_K \mathbf{F} \\ \vdots & \vdots & \ddots & \vdots \\ s_1(N-1) \mathbf{C}_1 \mathbf{F} & s_2(N-1) \mathbf{C}_2 \mathbf{F} & \cdots & s_K(N-1) \mathbf{C}_K \mathbf{F} \end{bmatrix} \]  

\[ \mathbf{s}_e = \begin{bmatrix} s_1(2) \mathbf{C}_1 \mathbf{F} & s_2(2) \mathbf{C}_2 \mathbf{F} & \cdots & s_K(2) \mathbf{C}_K \mathbf{F} \\ s_1(4) \mathbf{C}_1 \mathbf{F} & s_2(4) \mathbf{C}_2 \mathbf{F} & \cdots & s_K(4) \mathbf{C}_K \mathbf{F} \\ \vdots & \vdots & \ddots & \vdots \\ s_1(N) \mathbf{C}_1 \mathbf{F} & s_2(N) \mathbf{C}_2 \mathbf{F} & \cdots & s_K(N) \mathbf{C}_K \mathbf{F} \end{bmatrix} \]

\[ \mathbf{h} = [\mathbf{h}_1^1, \mathbf{h}_2^1, \cdots, \mathbf{h}_K^1, \mathbf{h}_1^2, \mathbf{h}_2^2, \cdots, \mathbf{h}_K^2]^T \]

The received signal model can be written as

\[ \mathbf{y} = \mathbf{Ah} + \mathbf{w} \]  

where \( \mathbf{y} \), \( \mathbf{A} \) and \( \mathbf{h} \) are \( NP \times 1 \), \( NP \times 2KL \) and \( 2KL \times 1 \) dimension matrices. To simplify the notation, \( \mathbf{h} \) is described as \( \mathbf{h} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_{2K}]^T \) for the following equations.

### III. Maximum-Likelihood (ML) Channel Estimation

Estimation of the channel impulse response vector is obtained by directly minimizing the following cost function

\[ \hat{\mathbf{h}} = \arg \min \{ ||\mathbf{y} - \mathbf{Ah}||^2 \} \]  

\[ \mathbf{h}_{\text{est}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{y} \]

where \( \mathbf{Q} \) is the matrix elements of \( \mathbf{Q} \) can be evaluated using the properties \( b_i^*(m)b_j(m) = 1 \), \( \mathbf{F}^T = \mathbf{F} \) and \( \mathbf{C}_k^T \mathbf{C}_k = \mathbf{I}_P \). It can be easily shown that in the case of \( i = j \), \( Q_{i,j} \) is equal to \( N L \). On the other hand, if \( (i \neq j) \) then \( Q_{i,j} \) is not identity matrix so the inverse of \( \mathbf{Q} \) matrix have to be calculated.
IV. ITERATIVE CHANNEL ESTIMATION

The problem of interest is the calculation of the inverse of the $2KL \times 2KL$ square matrix in (11). The inverse of $Q$ is of computational complexity $O((KL)^3)$ and requires significant computation for large values of $L$ and $K$. Especially, ongoing research with goal of increasing user capacity, the number of active user $K$ will be increased enormously. Therefore, instead of directly minimizing (9), the SAGE algorithm is proposed.

A. SAGE Algorithm

The SAGE algorithm is proposed by Fessler et al. [12] as a generalization of the EM algorithm. The suitable approach for applying the SAGE algorithm for the problem at hand is to decompose the received signal in (2) and (3) into the sum as follows

$$y = \sum_{k=1}^{2K} y_k$$

(12)

Here, $y_k$ represents the received signal component transmitted by $k$th user. Note that $y$ and $y_k$ in (12) are treated as the complete and the incomplete data respectively in the SAGE approach employed. By defining following equations,

$$x_e = \begin{bmatrix} s_k(1)C_k & 0 & \ldots & 0 \\ 0 & s_k(3)C_k & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & s_k(N-1)C_k \\ s_k(2)C_k & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & s_k(4)C_k \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & s_k(N)C_k \end{bmatrix}$$

(13)

$$x_e = \begin{bmatrix} x_e \\ x_e^* \end{bmatrix}$$

(14)

$$X_k = \begin{bmatrix} x_e & 0 \\ 0 & -x_e^* \end{bmatrix}$$

(15)

Equation (12) can be written in more succinct form as follows,

$$y_k = X_k \tilde{F}h_k + w_k \quad 1 \leq k \leq 2K$$

(16)

where $\tilde{F}$ is matrix which has dimension of $NP \times L$ in this form $\tilde{F} = [\mathbf{F}, \ldots, \mathbf{F}]$. The Gaussian noise vector, $w_k$ in (16) represents the portion of $w$ in the decomposition defined by $\sum_{k=1}^{2K} w_k = w$. Following the SAGE technique presented in [10], [11], the algorithm estimates the corresponding component in the received signal for each of the user links as follows,

**Initialization:** For $1 \leq k \leq 2K$

$$\hat{z}_k^{(0)} = X_k \tilde{F}h_k$$

(17)

At the $q$th iteration ($q=0,1,2,\ldots$): For $k = 1 + \lfloor q \mod 2K \rfloor$, compute

$$y_k^{(q)} = \tilde{z}_k^{(q)} + \left[ y - \sum_{k=1}^{K} \tilde{z}_k^{(q)} \right]$$

(18)

$$\hat{h}_k^{(q+1)} = \tilde{F}^\dagger X_k^{-1} y_k^{(q)}$$

(19)

For $1 \leq j \leq 2K$ and $j \neq k$

$$\tilde{z}_k^{(q+1)} = \hat{z}_k^{(q)}$$

(20)

In Eq.(21) $X_k$ is a diagonal matrix so the SAGE algorithm do not require any matrix inversion.

B. Initialization

Choosing initial values for the SAGE algorithm is an important issue for the convergence of speed of the algorithm. We can obtain an initial estimate of the channel for the SAGE-type iteration as follows:

$$\hat{h}_k^{(0)} = \tilde{F}^\dagger X_k^{-1} y.$$  

(22)

As expected from Eq.(22), increasing the number of active user will degrade the initialization performance of the algorithm. This in turn, will increase the number of iterations necessary for convergence as will be shown in the simulation section.

C. Complexity

In this subsection, we give a short description of the computational complexity of the proposed SAGE algorithm and ML estimator. The ML channel estimation given in (10) requires $4K^2L^2NP + 8K^3L^3 + 2KLN + 4K^2L^2$ complex multiplications. On the other hand, the computational load to implement the SAGE requires $4KNPL + 2KNP + q(2LN + 2NP)$ complex multiplication. Therefore, it is shown that the computational complexity of ML estimator higher than SAGE estimator. If we define the total numbers of multiplication required for the ML and SAGE algorithms as $ML_{Total}$ and $SAGE_{Total}$ respectively, then complexity rate can be summarized as given in Table 1. It is demonstrated that $N$ and $P$ values don’t affect the rate of $ML_{Total} \backslash SAGE_{Total}$ because these values have same effect on computational complexity of both ML and SAGE techniques. On the contrary, it is note that complexity rate between ML and SAGE is increased by the increasing number of $L$ and $K$.

<table>
<thead>
<tr>
<th>$L$</th>
<th>$K$</th>
<th>$N$</th>
<th>$P$</th>
<th>$q$</th>
<th>$ML_{Total}$</th>
<th>$SAGE_{Total}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>16</td>
<td>60</td>
<td>16</td>
<td>30</td>
<td>17.2</td>
<td>17.2</td>
</tr>
<tr>
<td>8</td>
<td>256</td>
<td>1000</td>
<td>512</td>
<td>80</td>
<td>1667.5</td>
<td>1667.5</td>
</tr>
<tr>
<td>16</td>
<td>256</td>
<td>1000</td>
<td>512</td>
<td>80</td>
<td>3467.3</td>
<td>3467.3</td>
</tr>
<tr>
<td>16</td>
<td>512</td>
<td>1000</td>
<td>512</td>
<td>80</td>
<td>7587.1</td>
<td>7587.1</td>
</tr>
<tr>
<td>16</td>
<td>512</td>
<td>500</td>
<td>512</td>
<td>80</td>
<td>7823.5</td>
<td>7823.5</td>
</tr>
<tr>
<td>16</td>
<td>312</td>
<td>2000</td>
<td>512</td>
<td>80</td>
<td>7470.1</td>
<td>7470.1</td>
</tr>
</tbody>
</table>

V. SIMULATIONS

In computer simulations, we assume that all the users’ signals are received with the same power($P_k=1$). The total transmitted power is normalized for the transmit antennas and orthogonal Walsh sequences selected as a spreading code for the users. The number of active users is equal to the length of the spreading code and to the number of subcarriers. Results
are compared in terms of MSE performance versus number of iteration. Each user sends its data frame composed of \( T \) pilot symbols, and \( F \) data symbols, over mobile frequency fading channel. Wireless channels between mobiles antennas and the receiver antenna are modeled based on a realistic channel model determined by COST-207 project in which Typical Urban(TU) channel model is considered having the channel length \( L \). The channel tap gains are given in Table 2.

QPSK signal modulation format is adopted with bandwidth is chosen as 1.228 MHz (Qual Comm-CDMA). We used the other parameters that is given in the first row of the Table.1. At the receiver, the initial ML channel estimate is obtained by using \( T \) preamble symbols.

The simulation results of uplink STBC-MC-CDMA system are demonstrated in form of curves. In the coming set of curves, we concentrate on two points: the effect of applying higher number of users and pilot tones. Fig. 2 and 3 display the MSE performance versus number of iteration, assuming equal SNR for all the \( K \) users. It is demonstrated that in Fig.2 and Fig.3 the SAGE algorithm converges the ML solution while its computational is lower than the ML estimation.

In particular, the number of increasing active users has been investigated in Fig.2 and it is shown that SAGE algorithm approach to ML channel estimator for \( K = 12 \) and \( K = 16 \) at 45 and 70 iteration, respectively. It is shown that performance of the ML channel estimator is degraded by the increasing the number of active user \( K \) because it needs to estimate more parameters \( h_k \) for constant pilot number \( T \). On the other hand, increasing the number of active user also affects the initialization of the SAGE algorithm. Therefore, the number of required iterations for the SAGE algorithm to converge the ML performance are also increase by the number of increasing active user. Therefore, in Fig. 3, the number of pilot tones \( T \) are increased to improve channel estimation performance for full load (\( K = 16 \)). It is concluded that MSE and the number of iteration are decreasing for the total number of increasing pilot tones.

VI. CONCLUSIONS

Maximum likelihood channel estimation for the uplink STBC-MC-CDMA systems is implemented by using the SAGE algorithm that is suitable for superimposed signals. It is shown that the proposed technique allows us to achieve the ML estimator performance, when direct computation of the matrix inversion is complex. The SAGE estimator allows us to have a simple reference system, with a rather low complexity, for further studies to propose solutions for the uplink of a realistic 4th generation mobile radio system. Moreover, it is
shown that the SAGE algorithm requires more iterations when the system capacity approaches to the full system capacity. It is demonstrated this problem could be solved by increasing the number of pilot tones.

REFERENCES


